# Magnetic moments of the $2_1^+$ states around $^{132}$Sn

B. A. Brown,[1] N. J. Stone,[2, 3] J. R. Stone,[2, 4] I. S. Towner,[5] and M. Hjorth-Jensen[6, 7, 8]

[1]*Department of Physics and Astronomy and National Superconducting Cyclotron Laboratory,*
*Michigan State University, East Lansing, MI 48824, USA*
[2]*Department of Physics, University of Oxford, Oxford, OX1 3PU, United Kingdom*
[3]*Department of Physics and Astronomy, University of Tennessee, Knoxville, TN 37996, USA*
[4]*Department of Chemistry and Biochemistry, University of Maryland, College Park, MD 20742, USA*
[5]*Physics Department, Queen's University, Kingston, Ontario, K7L 3N6, Canada*
[6]*Department of Physics and Center of Mathematics for Applications, University of Oslo, N-0316, Oslo, Norway*
[7]*PH Division, CERN, CH-1211 Geneva 23, Switzerland*
[8]*Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824, USA*
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Development of neutron-rich radioactive beams at the HRIBF facility has stimulated experimental and theoretical activity in heavy Sn and Te isotopes. Recently, the g-factor of the first $2^+$ state in $^{132}$Te has been measured. We report here new shell-model calculation of magnetic moments for selected Sn and Te isotopes. The residual interaction is based on the CD-Bonn renormalized $G$-matrix. Single-particle spin and orbital effective g-factors are evaluated microscopically including core polarization and meson exchange currents effects.

## I. INTRODUCTION

The g-factors of nuclear excited states yield valuable information on the make-up of their wavefunctions. The advent of radioactive beams (RIBs) constitutes a major new initiative in nuclear structure investigations, opening up many new experimental opportunities. Systematics of results, obtained with stable beams, can be extended to new areas of a special theoretical interest. The region of neutron-rich Sn and Te nuclei with the number of protons at or just above the Z=50 closed shell is now open to direct study using Coulomb excitation in inverse kinematics. The combination of a relative weak beam, low Coulomb excitation cross-section and rather high energy of the low-lying excited states, implies that only the first $2^+$ state is currently accessible to experiment. The B(E2; $0^+ \rightarrow 2^+$) values for $^{132,134,136}$Te were measured [1] using this technique. The results are in agreement with systematics and model calculations for $^{132,134}$Te (N=80,82). A rather unexpectedly low B(E2) value was obtained for $^{136}$Te. This anomaly is accompanied with a significant drop of $2^+$ state energies in the $N = 84$ nuclei $^{134}$Sn and $^{136}$Te compared to the $N = 80$ nuclei $^{130}$Sn and $^{132}$Te. In this way, both the first $2^+$ states and the $B(E2)$ values in N=80 and N=84 Sn and Te isotones are asymmetric with respect to $N = 82$. The g-factor of the first $2^+$ state in $^{132}$ Te has been reported recently [2].

Previous shell-model calculations (see Ref. [1] for details) provided reasonable agreement with energy spectra and B(E2) in N=80 and N=82 Sn and Te isotones but failed to explain the B(E2) value in $^{136}$Te. Magnetic moments were calculated for $^{134}$Te, $^{136,137}$Xe and $^{137}$Cs by Sakar and Sakar [3] with the KH5082 and CW5082 interactions fitted in the $^{208}$Pb and scaled to the $^{132}$Sn region, and with empirical effective single-particle g-factors. Shell-model calculations for the $2^+$, $4^+$ and

$6^+$ states in $^{130-134}$Te and $^{132-136}$Xe were reported in Ref. [4], where the surface delta interaction (SDI) was used with two different sets of parameters. The single particle states were chosen to reproduce single proton states in $^{133}_{51}$Sb and single neutron states in $^{131}_{50}$Sn. The single-particle spin and orbital effective $g$-factors were based on the experimental g-factors of the low-lying $(7/2)^+$ and $(5/2)^+$ states in the odd-Z, $N = 82$ isotones.

Properties of $2^+$ states around $^{132}$Sn have been also studied by Terasaki *et al.* [5] in a separable quadrupole-plus-pairing model. They investigated the nature and single-particle structure of the $2^+$ states and calculated B(E2, $0^+ \rightarrow 2^+$) values. The g-factors were presented for $^{134,136,138}$Xe and $^{132,134,136}$Te. Single-particle bare orbital g-factors and bare spin g-factors, multiplied by 0.7, were used in the calculation.

The present shell-model calculations were carried out in the proton-neutron formalism starting with $^{132}$Sn as a closed core. A realistic two-body residual interaction based on the CD-Bonn interaction model [6] was used. The magnetic moments take into account microscopic calculations of core-polarization and meson-exchange current effects.

## II. SHELL-MODEL HAMILTONIAN

The wave functions for $N \leq 82$ were obtained in the model space of $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})^{Z-50}$ for proton particles and $(0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})^{N-82}$ for neutron holes. Based upon the energy levels observed in $^{133}$Sb [7] the proton single-particle energies are $-9.68$, $-8.72$, $-7.24$, and $-6.88$ MeV for the $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$ and $0h_{11/2}$ orbitals, respectively. The proton $2s_{1/2}$ level is not yet observed and we use an estimated energy

TABLE I: Experimental energy levels up to 3 MeV for $^{134}$Te compared to the calculation with CD-Bonn.

| $J^\pi$ | Experiment | CD-Bonn |
|---|---|---|
| $0_1^+$ | 0.0 | 0.0 |
| $2_1^+$ | 1.28 | 1.21 |
| $4_1^+$ | 1.57 | 1.48 |
| $6_1^+$ | 1.69 | 1.61 |
| $6_2^+$ | 2.40 | 2.17 |
| $2_2^+$ | 2.46 | 2.45 |
| $4_2^+$ | 2.55 | 2.45 |
| $1_1^+$ | 2.63 | 2.41 |
| $3_1^+$ | 2.68 | 2.54 |
| $5_1^+$ | 2.73 | 2.54 |
| $0_2^+$ | | 2.66 |
| $3_3^+$ | 2.93 | 3.06 |

TABLE II: Experimental energy levels up to 3 MeV for $^{130}$Sn compared to the calculation with CD-Bonn.

| $J^\pi$ | Experiment | CD-Bonn |
|---|---|---|
| $0_1^+$ | 0.0 | 0.0 |
| $2_1^+$ | 1.22 | 1.38 |
| $7_1^-$ | 1.95 | 1.87 |
| $0_2^+$ | | 1.92 |
| $4_1^+$ | 2.00 | 2.08 |
| $2_2^+$ | 2.03 | 2.00 |
| $5_1^-$ | 2.08 | 2.02 |
| $4_2^+$ | | 2.08 |
| $4_1^-$ | 2.21 | 2.12 |
| $6_1^+$ | 2.26 | 2.28 |
| $0_3^+$ | | 2.34 |
| $8_1^+$ | 2.34 | 2.36 |
| $6_1^-$ | | 2.36 |
| $10_1^+$ | 2.43 | 2.42 |
| $2_3^+$ | | 2.41 |
| $5_1^-$ | | 2.57 |
| $1_1^+$ | | 2.58 |
| $6_2^-$ | | 2.62 |
| $3^-,4^+$ | 2.49 | |
| $4,5$ | 2.49 | |
| | 2.60 | |

of $-7.34$ MeV. Based upon the energy levels observed in $^{131}$Sn [8, 9] the neutron single-particle energies are $-9.74$, $-8.97$, $-7.31$, $-7.62$ and $-7.38$ MeV for the $0g_{9/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ and $0h_{11/2}$ orbitals, respectively. Note the energy of the $11/2^-$ state given at 242 keV in [8] has now been corrected to 65 keV in [9]. The wave functions for $N \geq 82$ were obtained with the same model space for protons as above and with a model space for neutrons of $(0h_{9/2}, 1f_{7/2}, 1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 0i_{13/2})^{N-82}$ with respective single-particle energies of -0.894, -2.455, -0.450, -1.601, -0.799 and 0.25 MeV.

We used the shell-model code OXBASH [10]. The residual two-body interaction is obtained starting with a $G$-matrix derived from the CD-Bonn [6] nucleon-nucleon interaction. The harmonic oscillator basis was employed for the single-particle radial wave functions with an oscillator energy $\hbar\omega$=7.87 MeV. The effective interaction for the above shell-model space is obtained from the $\hat{Q}$-box method and includes all non-folded diagrams through third-order in the interaction $G$ and sums up the folded diagrams to infinite order [11]. The Coulomb interaction was added to the interaction between protons. There are three parts to the Hamiltonian which will be considered in turn, the proton-proton ($pp$), neutron-neutron ($nn$) and proton-neutron ($pn$) interactions.

In [12] and [13] the $pp$ Hamiltonian based upon the Bonn-A $G$-matrix was used to study the N=82 isotones. The energy levels obtained with this Hamiltonian agreed with experiment to about a hundred keV. The present results for $^{134}$Te with CD-Bonn are shown in Table I; agreement with experiment is comparable to the previous results.

In [14] the $nn$ Hamiltonian based upon the CD-Bonn $G$-matrix was used to study the the Sn isotones. Our results for $^{130}$Sn were similar to these previous calculations, but differ due to the new energy for the $h_{11/2}$ level in $^{131}$Sn [9]. The results for $^{130}$Sn can be improved a

little by multiplying the $nn$ renormalized $G$-matrix by a factor of 0.90 - the spectrum for this adjusted interaction is shown in Table II. An adjustment of this magnitude is not unreasonable given that oscillator radial wavefunctions were used in the calculation of the $G$-matrix elements.

Finally we consider the $pn$ Hamiltonian. The key nucleus in this regard is $^{132}$Sb whose spectrum is determined entirely by the $pn$ interaction (together with the proton and neutron single-particle energies). Compared to the $pp$ and $nn$ Hamiltonians there has been little previous study of this interaction. In [15] Mach $et$ $al.$ report on results using a finite range effective interaction based upon [16]. The results with the CD-Bonn interaction are compared with experiment in Table III. The levels given in this table are a selected set of those observed in $\beta$-decay [17] and high-spin $\gamma$-decay [18]. The levels are labeled by their dominant theoretical component, although typically they are only about 80% pure.

Andreozzi $et$ $al.$ [19] have reported calculations using the Bonn-A $G$-matrix. Their method, however, differs in several respects from the one we use. They use isospin formalism with a $^{100}$Sn closed shell in contrast to our proton-neutron formalism starting with $^{132}$Sn as a closed shell core. The main drawback of the isospin formalism is that proton and neutron single-particle energies are not independent and do not reproduce the experimen-

TABLE III: Experimental energy levels (in MeV) for $^{132}$Sb compared to the calculation with CD-Bonn. Energies in square brackets are relative to that for the $8_1^-$ levels that is estimated to be near 0.20 MeV.

| $J^\pi$ | Experiment | CD-Bonn | Main Conf |
|---|---|---|---|
| $4_1^+$ | 0 | 0 | $\pi g_{7/2}$-$\nu d_{3/2}$ |
| $3_1^+$ | 0.08 | 0.12 | |
| $5_1^+$ | 0.16 | 0.26 | |
| $2_1^+$ | 0.43 | 0.52 | |
| $3_2^+$ | 0.53 | 0.56 | $\pi g_{7/2}$-$\nu s_{1/2}$ |
| $1_1^+$ | 1.32 | 1.56 | $\pi d_{5/2}$-$\nu d_{3/2}$ |
| $1_2^+$ | 2.27 | 2.27 | $\pi d_{7/2}$-$\nu d_{5/2}$ |
| $(8_1^-)$ | [0.20] | 0.24 | $\pi g_{7/2}$-$\nu h_{11/2}$ |
| $(6_1^-)$ | 0.25 | 0.32 | |
| $(5_1^-)$ | 0.39 | 0.35 | |
| $(4_1^-)$ | 0.48 | 0.47 | |
| $(9_1^-)$ | [1.22] | 0.84 | |
| $(10_1^+)$ | [3.00] | 2.93 | $\pi h_{11/2}$-$\nu h_{11/2}$ |
| $(11_1^+)$ | [3.40] | 3.28 | $\pi h_{11/2}$-$\nu h_{11/2}$ |

tal levels of $^{133}$Sb and $^{131}$Sn exactly. Another aspect is that the renormalization of the $G$-matrix is carried out only to second order. The result of including the 3rd order terms as we do is to make the $pn$ matrix elements somewhat larger. As also demonstrated in [11], the total interaction in the $T = 0$ channel to third order is of the order of 20% compared with the interaction up to second order, whereas in the $T = 1$ channel the difference is less than 10%. This is mainly due to the influence of the strong nuclear tensor force component stemming from the $^3S_1 - ^3D_1$ partial wave in the nucleon-nucleon interaction.

Finally as a test of the configuration mixing involving all components of the hamiltonian, we show the calculated levels for $^{132}$Te compared to experiment in Table IV. Experiment is taken from the new results of Hughes et al. [20]. The agreement between experiment and calculation is excellent. The $2^+$ states are in significantly better agreement with experiment than those given in the separable quadrupole-plus-pairing model of Terasaki et al. [5, 20]. In [20] $2^+$ assignments for the 2.25 and 2.36 MeV states were made on the assumption

that there should not be any $1^+$ states at this low excitation energy. However, our calculation predicts two $1^+$ states near 2.4 MeV, and on this basis the states observed in experiment should be labeled $(1,2)^+$. The dominant component of these $1^+$ states are related to the low-lying $1^+$ states in $^{134}$Te and $^{130}$Sn. With regard

TABLE IV: Experimental energy levels up to 2.5 MeV for $^{132}$Te compared to the calculation with CD-Bonn.

| $J^\pi$ | Experiment | CD-Bonn |
|---|---|---|
| $0_1^+$ | 0.0 | 0.0 |
| $2_1^+$ | 0.97 | 0.95 |
| $(2)_2^+$ | 1.66 | 1.64 |
| $4_1^+$ | 1.67 | 1.54 |
| $6_1^+$ | 1.77 | 1.68 |
| $0_2^+$ | | 1.70 |
| $(2)_3^+$ | 1.79 | 1.93 |
| $(7)_1^-$ | 1.92 | 1.88 |
| $(5)_1^-$ | 2.05 | 2.01 |
| $4_1^-$ | | 2.12 |
| $0_3^+$ | | 2.17 |
| $4_2^+$ | | 2.20 |
| $6_2^+$ | | 2.21 |
| $(2)_4^+$ | 2.25 | 2.25 |
| $1_1^+$ | | 2.36 |
| $1_2^+$ | | 2.41 |
| $6_1^+$ | | 2.43 |
| $(2)_5^+$ | 2.36 | 2.46 |
| $4_3^+$ | | 2.48 |

to the magnetic moment of the $2^+$ state discussed here we give the components of this wavefunction (those with probabilities of greater than one percent) in Table V. We can also decompose the $2^+$ wavefunction in terms of the coupling between the $^{134}$Te two-proton configuration $pp$ and the $^{130}$Sn two-neutron hole configuration $nn$. This coupling is dominated by two components, 48.9% for $pp(2^+)nn(0^+)$ and 32.1% for $pp(0^+)nn(2^+)$.

## III. MAGNETIC MOMENTS

The magnetic moment matrix element, expressed in terms of a reduced matrix element using the Wigner-Eckart theorem for an operator of rank $\lambda = 1$, is

$$< \omega J M = J \mid \hat{\mu} \mid \omega J M = J > = \begin{pmatrix} J & \lambda & J' \\ -J & 0 & J \end{pmatrix} < \omega J ||\hat{\mu}|| \omega J > \qquad (1)$$

TABLE V: Wavefunction components for the first $2^+$ state in $^{132}$Te (those greater than one percent).

| proton wavefunction | neutron wavefunction | probability |
|---|---|---|
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^8, (1d_{5/2})^6, (1d_{3/2})^2, (2s_{1/2})^2, (0h_{11/2})^{12}$ | 28.4 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^8, (1d_{5/2})^6, (1d_{3/2})^4, (2s_{1/2})^2, (0h_{11/2})^{10}$ | 21.0 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^8, (1d_{5/2})^6, (1d_{3/2})^3, (2s_{1/2})^1, (0h_{11/2})^{12}$ | 15.3 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^8, (1d_{5/2})^6, (1d_{3/2})^4, (2s_{1/2})^0, (0h_{11/2})^{12}$ | 8.2 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^8, (1d_{5/2})^4, (1d_{3/2})^4, (2s_{1/2})^2, (0h_{11/2})^{12}$ | 5.5 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^8, (1d_{5/2})^5, (1d_{3/2})^4, (2s_{1/2})^1, (0h_{11/2})^{12}$ | 4.3 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^6, (1d_{5/2})^6, (1d_{3/2})^4, (2s_{1/2})^2, (0h_{11/2})^{12}$ | 3.1 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^7, (1d_{5/2})^6, (1d_{3/2})^3, (2s_{1/2})^2, (0h_{11/2})^{12}$ | 2.8 |
| $(0g_{7/2})^2, (1d_{5/2})^0$ | $(0g_{7/2})^8, (1d_{5/2})^5, (1d_{3/2})^3, (2s_{1/2})^2, (0h_{11/2})^{12}$ | 1.8 |
| $(0g_{7/2})^0, (1d_{5/2})^2$ | $(0g_{7/2})^8, (1d_{5/2})^6, (1d_{3/2})^4, (2s_{1/2})^2, (0h_{11/2})^{10}$ | 1.1 |

The many-body reduced matrix element can be expressed as a sum of products over one-body transition densities (OBTD) times reduced single-particle matrix elements

$$< \omega J||\hat{\mu}||\omega J > = \sum_{k_\alpha k_\beta} \text{OBTD}(\omega J k_\alpha k_\beta \lambda) < k_\alpha||\mu||k_\beta >, \tag{2}$$

where the OBTD is given by

$$\text{OBTD}(\omega J k_\alpha k_\beta \lambda) = \frac{< \omega J||[a_{k_\alpha}^+ \otimes \tilde{a}_{k_\beta}]^\lambda||\omega J >}{\sqrt{(2\lambda+1)}}. \tag{3}$$

The sum is over all pairs of orbits for protons and neutrons that can couple up to a tensor of rank $\lambda = 1$.

The free-nucleon operator is defined as

$$\mu_{\text{free}} = g_l \mathbf{l} + g_s \mathbf{s}, \tag{4}$$

with $g_l(\text{proton}) = 1.0$, $g_l(\text{neutron}) = 0.0$, $g_s(\text{proton}) = 5.587$, $g_s(\text{neutron}) = -3.826$.

The magnetic moment operator in finite nuclei is modified from the free-nucleon operator due to core-polarization and meson-exchange current (MEC) corrections [21, 22]. The effective operator is defined as

$$\mu_{\text{eff}} = g_{l,\text{eff}} \mathbf{l} + g_{s,\text{eff}} \mathbf{s} + g_{p,\text{eff}}[Y_2, \mathbf{s}], \tag{5}$$

where $g_{x,\text{eff}} = g_x + \delta g_x$, $x = l$, $s$ or $p$, with $g_x$ the free-nucleon, single-particle $g$-factors ($g_p = 0$) and $\delta g_x$ the calculated correction to it. Note the presence of a new term $[Y_2, \mathbf{s}]$, absent from the free-nucleon operator, which is a spherical harmonic of rank $\lambda' = 2$ coupled to a spin operator to form a spherical tensor of multipolarity $\lambda = 1$.

The corrections, $\delta g_x$, are computed in perturbation theory for the closed-shell-plus-or-minus-one configuration with the closed shell being $^{132}$Sn. The first-order core-polarization correction involves coupling the valence nucleon to the $1^+$ particle-hole states: proton $(0g_{9/2}^{-1}, 0g_{7/2})$ and neutron $(0h_{11/2}^{-1}, 0h_{9/2})$. This term

leads to a large quenching in the $g_{s,\text{eff}}$ value but only a small change in $g_{l,\text{eff}}$. The calculation is easily extended to all orders in the RPA series, [21]. The residual interaction in these calculations is taken as a one-boson-exchange potential multiplied by a short-range correlation function. This modification is an approximate, but easy, way to obtain a $G$-matrix.

Meson-exchange current corrections arise because nucleons in nuclei are interacting through the exchange of mesons, which can be disturbed by the electromagnetic field. Since meson exchange involves two nucleons, the correction leads to two-body magnetic moment operators. In a closed-shell-plus-or-minus-one configuration, computation of this correction requires evaluation of the two-body matrix elements between the valence nucleon and one of the core nucleons, summed over all nucleons in the core. The results can be expressed in terms of an equivalent effective one-body operator, Eq. (5), acting on the valence nucleon alone. The details of the two-body MEC operators are described in [21] and updated in [23]. For consistency, the same mesons, coupling constants, masses and short-range correlations are used in the construction of the MEC operators as are used in the one-boson-exchange potential.

There are two further terms to consider. First is a mesonic correction in which the meson prompts the nucleon to be raised to an excited state, the $\Delta$-isobar resonance, which is then de-excited by the electromagnetic field. This correction leads to a two-body operator that is handled like the MEC correction. Second is a relativistic correction to the one-body operator, [21]. Both these corrections amount to only a few percent change to the magnetic moment, but are retained for completeness.

Finally there are other second-order core-polarization corrections not contained in the RPA series that are difficult to compute because there are no selection rules to limit the number of intermediate states to be summed. A further correction of the same order in meson-nucleon couplings is a core-polarization correction to the two-

body MEC operator. Fortunately, as Arima *et al.* [24, 25] have pointed out, the latter terms largely cancel the former. In our earlier work [26] this correction was not explicitly calculated, but effective $g$-factors from a comparable calculation in Pb were used. Here we have computed these terms, so our result differs a little from [26] but not significantly. The computation, however, was performed approximately. The closed shell was taken to be an $LS$-closed shell, with $A = 140$, and the computation performed in $LS$-coupling. This leads to a great saving in computation time and makes the calculation tractable However, the neutron excess orbitals are not now treated correctly. The intermediate-state summation is explicitly computed up $12\hbar\omega$ and geometrically extrapolated beyond that.

The resulting corrections to the $g$-factors from the sum of all these effects are listed in Table VI. It is evident that there is not a great deal of state dependence in the effective operator. Thus for orbitals not explicitly listed in Table VI, we have used average values for their effective $g$-factors: protons: $\delta g_l = 0.094$, $\delta g_s = -2.14$, $\delta g_p = 2.03$, while for neutrons: $\delta g_l = -0.039$, $\delta g_s = 1.92$, and $\delta g_p = -0.93$. All matrix elements have been evaluated with harmonic oscillator radial functions of characteristic frequency $\hbar\omega = 7.87$ MeV. Note that with a term $[Y_2, \mathbf{s}]$, in the effective magnetic moment operator, Eq. 5, there are non-zero off-diagonal matrix elements between $0g_{7/2}-1d_{5/2}$ and $1d_{3/2}-2s_{1/2}$ orbitals. These $l$-forbidden matrix elements are zero with the free-nucleon operator but non-zero here. However, their impact in the present calculation is very small. The results in Table VI strictly only apply to closed-shell-plus-or-minus-one configurations at a $^{132}$Sn closed shell. An occupation number-dependent effective operator was introduced in [27] to account for the effects of blocking in the core-polarization. However, for the nuclei considered here the occupation number dependence for the magnetic moments moments is not large (on the order of 0.02), and we do not include this effect.

The magnetic moment for the $7/2^+$ ground state of the $N = 82$ nucleus $^{133}$Sb is 3.00(1) [26] compared to the free-nucleon value of 1.717 and effective operator value of 2.925. Thus, as discussed in [26], the core-polarization and mesonic exchange corrections are essential for understanding the enhancement of this magnetic moment relative to its free-nucleon (Schmidt) value. Results discussed in [27] for other $N = 82$ nuclei $^{135}$I, $^{137}$Cs and $^{139}$La show the general importance of the effective operator for these more complicated configurations.

Experimental magnetic moments for low-lying excitations in a range of even-even isotopes close to $^{132}$Sn are compared in Table VII to those obtained with the free-nucleon $g$-factors.

## IV. DISCUSSION AND CONCLUSIONS

Examination of Table VII illustrates the quality of the present shell-model calculation. We stress that all the magnetic dipole moments are calculated in full model space and the effective $g$-factors are obtained from microscopic calculation. In comparison with the QRPA model of Terasaki *et al.* [5] we do not predict a dramatic decrease and change in sign of the magnetic moment of the $2_1^+$ state in $^{136}$Te which might have risen in that model as a consequence of overestimation of the contribution of neutron excitations to the total wavefunction. The experimental determination of this $g$-factor is clearly of great interest [29]. More generally, we find the contribution of the neutron components to magnetic dipole moment of low-lying states in Te, Xe and Ba isotopes much smaller than in Sn nuclei.

We see from Table VII that the calculated magnetic moments for the $2^+$ states have a maximum at $N = 82$ for the pure proton configuration. The main deviation between experiment and theory can be traced to the magnetic moments of the proton $2^+$ states for $N = 82$. The calculated moment with the effective operator for the $^{136}$Xe $2^+$ is about 15 percent larger than experiment. For $N = 80$ if we reduce the proton contribution by 15 percent we would get 0.82 for the $^{132}$Te $2^+$ moment and 0.69 for the $^{134}$Xe $2^+$ moment, in better agreement with the experimental values of 0.70(10) and 0.708(14), respectively.

In comparison with the previous shell-model calculations, we note that in the calculations of Jakob et al. [4] (the same model space as ours), the schematic SDI interaction was fitted to a set of single particle energies containing the old value of the $11/2^-$ state in $^{131}$Sn which is ~200 keV higher than the new value [9]. The levels schemes obtained with the SDI interaction are not shown in [4]. The effective magnetic moment operator in [4] was obtained from a fit of $g$-factors in odd-even nuclei. The values obtained are close to our microscopic results, but the fitted operator does not include the tensor-type correction obtained microscopically. The calculations of Sarkar and Sarkar [3] are based on a fitted interaction for N=82 plus a renormalized $G$- matrix extrapolated from the region of $^{208}$Pb for N>82. Their wavefunctions for N=82 should be comparable in accuracy to the present model. Again however, an effective magnetic moment operator used in the evaluation of magnetic moments is fitted to experimental data and does not include the tensor-type term.

In summary, our microscopic interaction is based on a realistic nucleon-nucleon interaction and yields an excellent agreement with experiment for the energy levels of nuclei near $^{132}$Sn. This is the first fully microscopic calculation, using effective $g$-factors obtained from calculated core-polarization and mesonic exchange corrections. The good agreement with experiment confirms not only the validity of the shell model hamiltonian but also of the microscopic effective $g$-factors. Significant predic-

TABLE VI: Effective $g$-factors from core-polarization and MEC calculations.

| | Proton | | | | Neutron | | |
|---|---|---|---|---|---|---|---|
| Orbital | $\delta g_l$ | $\delta g_s$ | $\delta g_p$ | | $\delta g_l$ | $\delta g_s$ | $\delta g_p$ |
| $0h$ | 0.087 | −1.988 | 1.549 | | −0.033 | 1.847 | −0.769 |
| $0g$ | 0.131 | −2.284 | 1.705 | | −0.067 | 2.033 | −0.591 |
| $1d$ | 0.063 | −2.167 | 1.681 | | −0.018 | 1.976 | −1.039 |
| $2s$ | | −2.102 | | | | 1.804 | |
| $0g-1d$ | | | 3.329 | | | | −1.162 |
| $1d-2s$ | | | 1.902 | | | | −1.093 |

TABLE VII: Experimental and calculated magnetic moments. The calculations use the free-nucleon $g$-factors and effective $g$-factors. The last two columns give the proton and neutron contributions to the effective operator moments. Results for $4^+$ and $6^+$ states are added for comparison to related experiment data.

| Nuclide | $n$ | $J^\pi$ | Experiment | Effective | Free | proton | neutron |
|---|---|---|---|---|---|---|---|
| $^{124}$Sn | 74 | $2^+$ | −0.3(2)[28] | −0.270 | −0.364 | 0 | −0.270 |
| $^{126}$Sn | 76 | $2^+$ | | −0.262 | −0.355 | 0 | −0.262 |
| $^{128}$Sn | 78 | $2^+$ | | −0.253 | −0.343 | 0 | −0.253 |
| $^{130}$Sn | 80 | $2^+$ | | −0.275 | −0.385 | 0 | −0.275 |
| $^{134}$Sn | 84 | $2^+$ | | −0.469 | −0.745 | 0 | −0.469 |
| $^{130}$Te | 78 | $2^+$ | 0.59(7)[4] | 0.693 | 0.360 | 0.806 | −0.113 |
| $^{132}$Te | 80 | $2^+$ | 0.70(10)[2] | 0.975 | 0.575 | 1.027 | −0.052 |
| $^{132}$Te | 80 | $4^+$ | | 3.18 | 1.90 | 3.20 | −0.02 |
| $^{132}$Te | 80 | $6^+$ | 5.08(15)[4] | 5.14 | 3.20 | 5.15 | −0.01 |
| $^{134}$Te | 82 | $2^+$ | | 1.724 | 1.035 | 1.724 | 0 |
| $^{134}$Te | 82 | $4^+$ | | 3.44 | 2.04 | 3.44 | 0 |
| $^{134}$Te | 82 | $6^+$ | 4.7(6)[4] | 5.20 | 3.15 | 5.20 | 0 |
| $^{136}$Te | 84 | $2^+$ | | 0.695 | 0.544 | 0.846 | −0.151 |
| $^{134}$Xe | 80 | $2^+$ | 0.708(14)[4] | 0.825 | 0.541 | 0.886 | −0.061 |
| $^{136}$Xe | 82 | $4^+$ | 3.2(6)[28] | 3.55 | 2.17 | 3.55 | 0 |
| $^{136}$Xe | 82 | $2^+$ | 1.53(9)[4] | 1.823 | 1.165 | 1.823 | 0 |
| $^{138}$Xe | 84 | $2^+$ | | 0.775 | 0.623 | 0.912 | −0.137 |
| $^{138}$Ba | 82 | $2^+$ | 1.44(22)[28] | 2.00 | 1.52 | 2.00 | 0 |

tions are made to stimulate further experimental $g$-factor measurements in this region.

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